Universal Turing Machines

Lecture 30
Section 10.4

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Outline

1. Universal Turing Machines
2. Enumerators
3. Assignment

Universal Turing Machines

Definition (Universal Turing Machines)

A universal Turing machine is a Turing machine that can simulate any Turing machine.

- The input tape contains two items:
  - An encoding of the Turing machine M to be simulated.
  - The input w to be read by M.
- The universal Turing machine U will read M and w and
  - Accept, reject, or loop, according to what M would do when reading w,
  - Write to the tape the output that M would write when reading w.

Universal Turing Machines

- The encoding is straightforward:
  - The states $\{q_1, q_2, \ldots, q_n\}$ are encoded as $1, 11, \ldots, 111\cdots1$ and separated by 0's.
    - q_1 is the start state.
    - q_2 is the sole final state.
  - The tape symbols $\{a_1, a_2, \ldots, a_m\}$ are encoded as $1, 11, \ldots, 111\cdots1$ and separated by 0's.
  - "Left" is encoded as 1.
  - "Right" is encoded as 11.
- For example, the transition $\delta(q_2, a_3) = (q_1, a_4, R)$ would be encoded as

$$11011101011110 11$$

- The universal Turning machine U has three tapes.
  - Tape 1 contains the encoding of a machine M.
  - Tape 2 contains the encoded input to M.
  - Tape 3 contains the current state.

Universal Turing Machines

- U reads the current state q_i from Tape 3 and the current symbol a_i from Tape 2.
- It searches Tape 1 for a transition $(q_i, a_j) = (q_k, a_m, d)$.
- When it finds it, it
  - Replaces q_i with q_k on Tape 3.
  - Replaces a_j with a_m on Tape 2.
  - Moves left or right on Tape 2, according to d.
- If and when it fails to find (q_i, a_j) on Tape 1, it quits.

Outline

Enumeration

Definition (Enumerate)

To enumerate a language L is to list all of the strings in L, each string listed exactly once. The order does not matter.

Definition (Enumerator)

An enumerator for a language L is a Turing machine that enumerates L on its tape.

- If L is infinite, then clearly the enumerator does not halt.

Enumerators

Example (Enumerator)

- Let $\Sigma = \{0, 1\}$.
- It is easy to build an enumerator for Σ^*.
  - Begin with \$ on the tape.
  - Write λ "on the tape."
  - Move right, write a separator \$, and write 0.
  - Copy the last number written and increment the copy, unless the copy is all 1's, in which case replace it with all 0's and one additional 0.
  - Write the separator \$.
  - Repeat the previous three steps.
- The tape contents will be

$$\$\$0\$1\$00\$01\$10\$11\$000\ldots$$

Enumerators

- Let $\Sigma = \{0, 1\}$.
- Describe enumerators for the following languages.
  - All strings with an equal number of 0's and 1's.
  - All legitimate patterns of parentheses, where 0 represents '(' and 1 represents ')'.
  - All prime numbers.
  - All numbers that divide one of the decimal integers $9, 99, 999, 9999, 99999, \ldots$
  - All pairs $\{a, b\}$, where $a, b \geq 0$.
  - All triples $\{a, b, c\}$, where $a, b, c \geq 0$.

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Assignment

Homework

- Section 10.3 Exercises 1, 2, 4, 5, 6, 8, 9, 10.